NUMERICAL AND SIMULATION TECHNIQUES IN FINANCE

Edward D. Weinberger, Ph.D., F.R.M
Adjunct Assoc. Professor
Dept. of Finance and Risk Engineering
edw2026@nyu.edu
Office Hours by appointment

This half-semester course introduces the vast body of knowledge about how to actually implement various financial calculations on a digital computer. Much has been made about the enormous increases in calculating speed that have been achieved by computing hardware in the past few decades, but what is much less widely known is that there have been improvements of similar magnitude in the numerical algorithms that run on that hardware.

Only a small portion of this material can reasonably be presented in a half-semester course, so I have tried to focus on those aspects of it that are either directly relevant to financial engineering practice or helpful in understanding the directly relevant aspects. Thus, this course will include

- Solutions to non-linear algebraic equations found in finance
- Solution of ordinary differential equations and (if time permits) stochastic differential equations
- Solution of the partial differential equations of quantitative finance
- Numerical integration (“quadratures”), including the fast Fourier transform and an introduction to Monte Carlo methods
- Interpolation, most notably splines and “least squares”
- A survey of optimization techniques

Prerequisites:

FRE6083 (Quantitative Methods) or equivalent and graduate standing. If you have these, you will have an understanding of multivariate calculus, a basic course in probability, and some prior study of both ordinary and partial differential equations.

Students will be expected to write programs in EXCEL/VBA (see “Grading” below for details). Previous experience has shown that students who have written code in C, C++, java, or Python have little trouble picking up EXCEL/VBA. *However, students with no prior programming experience, or only experience running “canned” programs in languages such as MatLab, may well have trouble with this course.*

Required text:

Numerical Recipes: The Art of Scientific Computing by Frank Press, *et. al.*,
3rd Ed., Cambridge University Press, 2007, ISBN-10: 0521880688. The 2nd edition is available free at http://www.nrbook.com/a/bookcpdf.php, but this is such an outstanding book that it’s worth investing in the latest and greatest. References to chapters below assume the 3rd edition.

Recommended Reading

Options, Futures, and Other Derivatives, J. Hull, Prentice-Hall. Upper Saddle River, New Jersey. Any edition will do; hence the omission of ISBN numbers.

Introduction to Numerical Analysis, 3rd Ed., W. Stoer and R. Bulirsch, Springer-Verlag, New York. ISBN-10: 038795452X

This is the definitive book on much of numerical analysis. However, it is translated from the German, and it reads like it!

Various books by Paul Wilmott and co-authors

Grading:

Grades will be assigned based on the preparation of two programming assignments in EXCEL/VBA outside of class. The assignments are as follows:

<u>Project I</u>

Code and test a Visual Basic function to compute the implied volatility of an American option that does not pay dividends.

<u>Project II</u>

Code and test a Visual Basic function to perform a Value-at-Risk calculation of a given equities portfolio.

Detailed Course Outline:

<u>Introduction, by way of Root Finding and Ordinary Differential Equations</u>

I “Machine numbers” vs standard mathematical numbers
- A) “short” and “long” integers
- B) Radix and mantissa of floating point numbers
- C) The “machine epsilon”
- D) Round-off error examples
- E) Machine independent measures of computational “work”

II Root finding in one dimension – implied volatility of European Call
- A) Bisection
- B) Secant method
- C) Newton’s method
- D) Brent’s method

III Newton’s method in *N* dimensions

IV Ordinary Differential Equations (ODE’s)

A) Existence theorem
B) Initial value problems vs boundary value problems
C) Euler's method, explicit and implicit
D) Runge-Kutta
E) Variable Stepsizes

Reading: Chapter 1.1, 9, and 17 of *Numerical Recipes*

Partial Differential Equations – One Space/Price Dimension

I Intro. to partial differential equations (PDE's)
- A) Types of PDE's
  - i) Cauchy problems vs boundary value problems
  - ii) Hyperbolic, Elliptic, and Parabolic
  - iii) Free boundary problems
- B) Heat/diffusion equation as prototypical parabolic PDE
- C) Analytic solutions
- D) Standard finite difference approaches
  - i) Forward Time, Centered Step (FTCS)
  - ii) Von Neumann stability and lack thereof
  - iii) Fully implict methods
  - iv) Crank Nicholson
- F) Dealing with sparse matricies
- G) Binary and Trinary Trees – Accuracy vs Computational Effort
  - i) Binary tree for European/American option w/ error estimate
    - (1) Hull method
    - (2) CRR method
    - (3) Discrete dividends
    - (4) Computing Greeks
  - ii) Trinary trees

Reading: Chapter 20, Sections 0, 2, and 3 of *Numerical Recipes*
(Covers both lectures on partial differential equations)
Numerical methods chapters of Hull (Chapter number varies by edition)

Partial Differential Equations – Multiple Space/Price Dimensions

I Example of how multiple space/price dimensions arise in finance
II Discretization of the multiple dimensional diffusion operator
III Finite difference approaches and Von Neumann Stability revisited
- A) Forward Time, Centered Step (FTCS)
- B) Fully implict methods
- C) Crank Nicholson

IV Jacobi method and Successive Over-Relaxation (SOR)
V Operator splitting methods

Numerical Integration (Numerical Quadratures)

I Basic methods, inc. Gaussian quadrature

Reading: *Numerical Recipes*, Chapter 4, though the Wikipedia account of Gaussian quadrature, http://en.wikipedia.org/wiki/Gaussian_quadrature, is at least as helpful

II The Fast Fourier Transform (Reading: *Numerical Recipes*, Chapter 12)

Reading: *Numerical Recipes*, Chapter 12, though the Wikipedia account of the Fast Fourier transform, http://en.wikipedia.org/wiki/Fast_Fourier_transform, is at least as helpful. I will be presenting the Cooley-Tukey version, so pay special attention to that.

Monte Carlo Simulation (as much of the following as time permits)

- I Random number generation: An Oxymoron, but a Useful One
- II Monte Carlo with Clever Tricks for Variance Reduction
  - A) The efficient market hypothesis as a rationale for Monte Carlo
  - B) Finding the area of a circle: a simple Monte Carlo calculation
    - i) Statistical analysis
    - ii) Random (or not) number generation
  - C) Non-uniform random numbers
  - D) Generating correlated random variables
  - E) Variance reduction techniques
    - i) Importance sampling
    - ii) Antithetic variance applied to Black Scholes European Call
    - iii) Control variates and stratified sampling
  - F) Monte Carlo methods for American options
- III Low discrepancy sequences
  - A) The most basic low discrepancy sequence is the Halton sequence
  - B) The more sophisticated Sobol' sequence seems to work better

Reading: *Numerical Recipes*, Chapter 7.0-7.3, 7.6, though *Probability, Random Variables, and Stochastic Processes* by Athanasios Papoulis (a Poly prof!) has a better explanation of how non-uniform random variables can be generated from uniform ones.

Linear and Spline Interpolation

- I Why polynomial and linear interpolation don't cut it
- II Splines
  - A) "Natural" splines
  - B) B-splines
- III Limitations of splines
- IV Two ways of improving on standard splines
  - A) Rational interpolation
  - B) Splines with tension

V “Least squares” a.k.a. multiple regression

Numerical Recipes Chapter 3 of *Numerical Recipes* has a discussion of splines, but I find both the Wikipedia article on splines, http://en.wikipedia.org/wiki/Spline_interpolation, and the Wolfram math world article on splines, http://mathworld.wolfram.com/CubicSpline.html easier to understand.

Optimization in one and several dimensions

I Why optimization important in finance
II Example: Max. likelihood estimation of GARCH(1,1) model
III Some unconstrained optimization problems and techniques
  A) Markowitz optimization and the CAPM
  B) “Lin min”
  C) Nelder & Mead’s Simplex method
  D) Fletcher Powell
IV Constrained optimization
  A) Linear programming
  B) Constrained quadratic optimization and the Black-Litterman model
V Combinatorial optimization (*e.g.* The Traveling Salesman Problem; Markowitz optimization with constraints)
VI The Levenberg Marquant method

Reading: *Numerical Recipes*, Chapter 10, esp. 10.1 thru 10.5

Simulating Stochastic Differential Equations (SDE’s) (time permitting)

I What SDE’s actually are - stochastic calculus background
  A) Modes of stochastic convergence
  B) Ito’s lemma
  C) The Ito integral as the l.i.m. of a stochastic sum
II Example SDE: The lognormal stock price process
III The Euler-Maruyama method
IV Convergence modes of method
  A) Strong convergence: convergence of “mean of error”
  B) Weak convergence: convergence of “error of mean”
  C) Long term stability
VI The Milstein method

Reading: Hingam’s Introduction to Numerical Solution of SDE’s,