B.Sc. Physics

First Semester

Mechanics and Properties of Matter

Unit 1: Frames of Reference and Gravitation: **11 hours**

Frames of Reference: Inertial frames, Galilean transformation equations – position, velocity & acceleration, non-inertial frames of reference. Concept of Fictitious force, rotating frame of reference – relation between acceleration in inertial frame and rotating frame, concept of Coriolis force.

Gravitation: Newton's Law of Gravitation, Kepler's laws of planetary motion (derivation). Principle of launching of satellites, expressions for orbital velocity, period & altitude of satellites (derivation). Escape velocity (derivation), Geostationary satellites (brief). Remote Sensing Satellites (brief explanation & applications)

Frames of Reference

In order to describe the motion of a body we should know, with respect to what the motion has been measured.

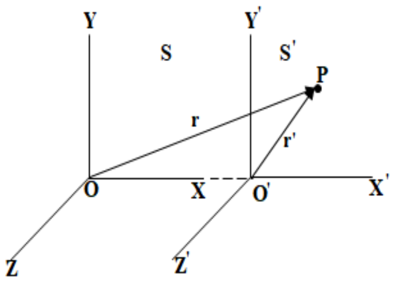
A system of co-ordinate axes which defines the position of a particle or an event in two- or three-dimensional space is called a *frame of reference*. The essential requirement of a frame of reference is that it should be rigid.

The simplest frame of reference is a Cartesian co-ordinate system. In this system the position of a particle at any point of its path is given by three co-ordinates (x, y, z).

Inertial Frame of Reference:

Those frames of reference in which Newton's first and second laws hold good are called *inertial frames of reference*. In such a frame, if a body is not acted by external force, it continues in its state of rest or uniform translatory motion and hence they are called inertial frames.

Thus, in an inertial frame, if a body is not experiencing any external force, its acceleration 'a' is given by



$$a = \frac{d^2r}{dt^2} = 0$$

$$where\ r = x\hat{\imath} + y\hat{\jmath} + z\hat{k},$$

$$called\ position\ vector\ drawn\ from\ the\ origin.$$

$$\frac{d^2x}{dt^2} = 0, \frac{d^2y}{dt^2} = 0, \frac{d^2z}{dt^2} = 0$$

Let us consider an inertial frame 'S' and another frame 'S`', which is moving with constant velocity 'v' relative to S. initially at t=0, if the positions of the origins of the two frames coincide, then in the two frames the position vectors of any particle 'P' at any instant 't' can be related by the following expressions.

$$r = OO' + r'$$

$$r = vt + r'$$

$$r' = r - vt$$

$$\frac{dr'}{dt} = \frac{dr}{dt} - v$$

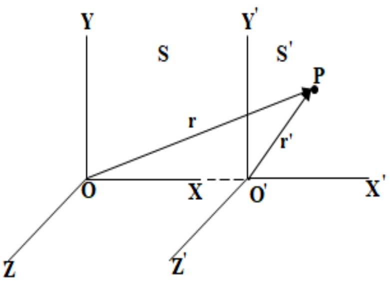
As *v* is constant,

$$\frac{d^2r'}{dt^2} = \frac{d^2r}{dt^2} \ or \ \boldsymbol{a}' = \boldsymbol{a}$$

That is, a particle experiences the same acceleration in two frames, which are moving with constant relative velocity. Thus, one can conclude that if S frame is an inertial frame, then all those frames which are moving with constant velocity relative to the first frame are also inertial. Thus, it is clear that inertial frames are non-accelerating frames and also non-rotating frames.

Galilean Transformation:

A point or a particle at any instant, in space has different co-ordinates in different reference systems. The equations which provide the relationship between the co-ordinates of two reference systems are called transformation equations. The transformation of co-ordinates of a particle from one inertial frame to another is known as *Galilean transformations*.



We know that a frame S', which is moving with constant velocity v relative to an inertial frame relative to an inertial frame S, is itself inertial. If we assume

i. that the origins of the two frames coincide at $t = 0$.

ii. that the co-ordinate axes of the second frame are parallel to that of the first and

iii. that the velocity of the second frame relative to the first is v along X-axis, then the position vectors of a particle at any instant are related by the equation,

$$r = r' + vt$$

$$r' = r - vt$$

In the component form, the co-ordinates are related by the equations.

$$x' = x - vt \; ; \; y' = y \; ; \; z' = z$$

The above equation expresses the transformation of co-ordinates from one inertial frame to another and hence they are referred as Galilean transformations.

Now if we assume time to be independent of any frame of reference, we have the four transformation equations from S to S'. That is

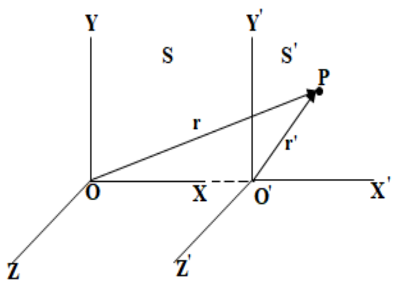
$$x' = x - vt \; ; \; y' = y \; ; \; z' = z \text{ and } t' = t.$$

The inverse transformations (from S' to S) are

$$x = x' + vt' \; ; y = y' \; ; z = z' \;\; \text{and } t = t'.$$

Transformation of Position:

Consider frame S' moving with constant velocity v relative to frame S. Consider a particle at P. Let 'r' be the position vector of the particle with reference to frame S and r' be the position vector of the particle with reference to the frame S'. The position of the particle as measured by the observer from the frame S is given by



$$r = r' + vt$$

$$r' = r - vt$$

In the component form, the co-ordinates are related by the equations.

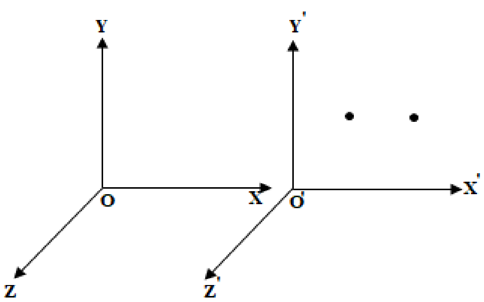
$$x' = x - vt$$

$$y' = y$$

$$z' = z$$

The above equations are the position of the particle under Galilean transformations.

Transformation of velocity:



Let us now consider the velocity measurements $u \text{ and } u'$ as made by the two observers in their respective frames of reference. For this, we consider a displacement in the $+x$ direction, say and t the time interval for it. Thus, if $x_1, t_1 \text{ and } x_2, t_2$ be the co-ordinates of the initial and final events characterizing the displacement in frame S. x_1', t_1' and x_2', t_2' the corresponding co-ordinate in frame S', we have

Velocity relative to frame S is given by

$$u = \frac{x_2 - x_1}{t_2 - t_1} \ldots \ldots . . (1) \quad \&$$

Velocity relative to frame S' is given by

$$u' = \frac{x'_2 - x_1'}{t_2' - t_1'} \ldots \ldots \ldots (2)$$

Since

$$x_2' = (x_2 - vt_2):\ x_1' = (x_1 - vt_1):\ t_1' = t_1:\ t_2' = t_2$$

$$u' = \frac{(x_2 - vt_2) - (x_1 - vt_1)}{t_2 - t_1}$$

$$u' = \frac{(x_2 - x_1) - (t_2 - t_1)v}{t_2 - t_1}$$

$$u' = u - v$$

That is velocities measured by the observers in the two frames of reference, are not same. We therefore say that velocity is not invariant under Galilean transformations.

Transformation of Acceleration:

Consider accelerations a and a', as measured by the observer in their respective frames of reference, we have

$$a = \frac{du}{dt} \ and \ a' = \frac{du'}{dt}$$

Since

$$u' = u - v$$

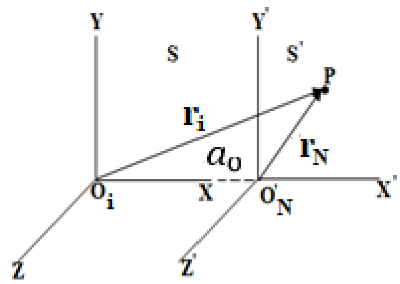
$$\frac{du'}{dt} = \frac{du}{dt} - 0$$ v being constant

$$\therefore\ a' = a$$

That is, the acceleration as measured by the two observers in the two frames of reference respectively are the same. Acceleration is thus invariant to Galilean transformations.

Non-Inertial frames and fictitious forces:

Frames of references in which Newton's law of inertia does not hold are called *non-inertial frames*. All the accelerated and rotating frames are the non-inertial frames of reference.



According to Newton's second law, the applied force F on a body of mass 'm' is given by

$$F_i = ma_i \text{-----------} (1)$$

Where a_i is the acceleration observed in an inertial frame. This equation is not valid for accelerated frames because due to its own acceleration, the accelerated observer will observe a_N and hence,

$$F_i \neq ma_N$$

If no external force is acting on a particle, even then in the accelerated frame it will appear that a force is acting on it. Suppose that S is an inertial frame and another frame S' is moving with an acceleration a_0 relative to S. The acceleration of a particle P on which no external force is acting, will be zero in the frame S, but in frame S' the observer will find that an acceleration $-a_0$ is acting in it. Thus, in frame S', the observed force on the particle is $-ma_0$.

Such a force, which does not really act on the particle but appears due to the acceleration of the frame is called a fictitious or pseudo force. Hence fictitious force on the particle P is

$$F_0 = -ma_0$$

Let S be an inertial frame of reference and S' be a non-inertial frame, suppose frame S' coincides at $t = 0$ with S. then at any time t, r_i *and* r_N are the position vectors in inertial and non-inertial frames. Then,

$$r_i = r_N + O_iO_N$$

$$r_i = r_N + \frac{1}{2}a_0t^2 \text{-----------} (1)$$

Where a_0 is the acceleration of the frame S' relative to S. Differentiating equation 1 w.r.t 't', we get

$$\frac{dr_i}{dt} = \frac{dr_N}{dt} + a_0t \text{-------} (2)$$

Differentiating equation 1 w.r.t 't', we get

$$\frac{dr_i^2}{dt} = \frac{dr_N^2}{dt} + a_0$$

$$a_i = a_N + a_0$$

$$a_i - a_0 = a_N$$

$$ma_i - ma_0 = ma_N$$

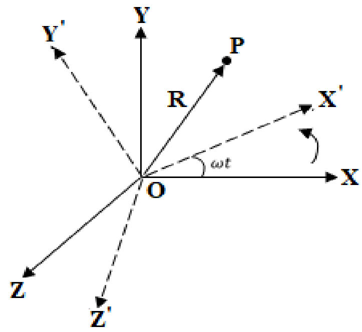
$$\therefore \quad F_i + F_0 = F_N$$

Examples:

1. Consider a particle at rest with respect to a lift which is descending with an acceleration equal to g, if 'm' be the mass of the particle, the fictitious force acting on it is $F_0 = -mg$. So that, the resultant force on it as observed by an observer in the lift is given by $F_N = F_i + F_0 = mg - mg = 0$.that is the particle is weightless and thus remains suspended in the air, a condition that obtains in artificial satellites.
2. Suppose that a bucket, half filled with water is stationary in an inertial frame S. in this condition the water surface is flat but in the rotating frame S' the water surface is seen parabolic (depression at the centre). This is due to the fact n the non-inertial frame a fictitious force acts in the water.

Rotating frame of reference:

'Coriolis force is a fictitious force which acts on a particle only if it is in motion with respect to the rotating frame'.



Consider a reference frame S' rotating with a uniform angular velocity ω with respect an inertial frame S. let us assume the two frames of reference have common origin O. the position vector of a particle P in both frames will be the same. That is $R_i = R_r = R$, because the origins are coincident. Now, if the particle P is stationary in the frame S, the observer in the rotating frame S' will see that the particle is moving oppositely with linear velocity $-\omega \times R$. Thus, if the velocity of the particle in the frame S is $\left(\frac{dR}{dt}\right)_i$, then its velocity $\left(\frac{dR}{dt}\right)_r$ in the rotating frame will be given by

$$\left(\frac{dR}{dt}\right)_r = \left(\frac{dR}{dt}\right)_i - \omega \times R$$

$$\left(\frac{dR}{dt}\right)_i = \left(\frac{dR}{dt}\right)_r + \omega \times R \dots\dots\dots (1)$$

This equation holds for all vectors and relates the time derivatives of a vector in the frame S and S'. Therefore relation (1) may be written in the form of operator equation as

$$\left(\frac{d}{dt}\right)_i = \left(\frac{d}{dt}\right)_r + \omega \times \dots\dots\dots (2)$$

Writing $\frac{dR}{dt} = v$ for the velocity of the particle, we have

$$v_i = v_r + \omega \times R \dots\dots\dots(3)$$

Now, if we operate equation (2) on velocity vector v_i, we have,

$$\left(\frac{dv_i}{dt}\right)_i = \left(\frac{dv_i}{dt}\right)_r + \omega \times v_i$$

Substituting the value of v_i in the righthand side of the above equation from equation (3), we get.

$$\left(\frac{dv_i}{dt}\right)_i = \left(\frac{d}{dt}(v_r + \omega \times R)\right)_r + \omega \times (v_r + \omega \times R)$$

$$\left(\frac{dv_i}{dt}\right)_i = \left(\frac{dv_r}{dt}\right)_r + \frac{d\omega}{dt} \times R + \omega \times \left(\frac{dR}{dt}\right)_r + \omega \times v_r + \omega \times (\omega \times R)$$

$$a_i = a_r + 2\omega \times v_r + \omega \times (\omega \times R) + \frac{d\omega}{dt} \times R$$

In the case of earth, ω is constant, so $\frac{d\omega}{dt} = 0$. Then

$$a_i = a_r + 2\omega \times v_r + \omega \times (\omega \times R) \dots\dots\dots(4)$$

If m is the mass of the particle, then force in the rotating frame is

$$ma_i = ma_r + 2m\omega \times v_r + m\omega \times (\omega \times R)$$

$$ma_r = ma_i - 2m\omega \times v_r - m\omega \times (\omega \times R)$$

$$ma_r = F_i + F_0$$

$$Fictitious\ force\ F_0 = -2m\omega \times v_r - m\omega \times (\omega \times R)$$

Where $-2m\omega \times v_r$, is the Corioli's force and $-m\omega \times (\omega \times R)$, the centrifugal force. Hence, in the rotating frame if a particle moves with velocity v_r, then it always experiences a Coriolis force. Its direction is always perpendicular to that of v_r.

Effect of Coriolis force:

In case a body is in motion relative to the rotating frame of reference of the earth, the fictitious Coriolis force comes into play and two cases arise.

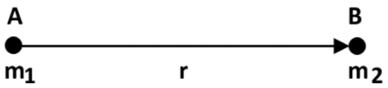
i. When the body is dropped from rest so as to fall freely under the action of the gravitational force: - The horizontal component of the Coriolis force acting on the freely falling body deflects it a little from its truly vertical path. The vertical component obviously produces no such deflection but only affects the value of ‘g’.

ii. When it is given a large horizontal velocity: - If the horizontal velocity of the body be sufficiently large, so that it covers fairly large horizontal distances, the Coriolis force causes a moving particle in the northern hemisphere to deflect towards the right of its path. In the southern hemisphere the deflection is towards the left of the path.

Gravitation

Newton’s Law of Universal Gravitation:

According to this law, “*Everybody in this universe attracts every other body with a force which is directly proportional to the product of their masses and inversely proportional to the square of the distance between their centers*”.



Consider two bodies, A and B, of masses m_1 and m_2 respectively, with r being the distance between their centers. Then, the force of gravitational attraction between them is,

$$F \propto \frac{m_1 m_2}{r^2}$$

$$F = \frac{G\, m_1 m_2}{r^2}$$

Where, G is a universal constant of gravitation. If $m_1 = m_2 = 1Kg$ and $r = 1m,$ then, $F = G.$ Thus, gravitational constant can be defined as “It is numerically equal to the force of attraction between two-unit masses with their centers separated by unit distance”. The value of G is $6.67 \times 10^{-11}\ Nm^2Kg^{-2}$ for all pair of bodies.

Kepler’s Laws of Planetary Motion:

Summarizing the whole observational data, collected by the ancient astronomers, available till then, Kepler formulated three laws, which describes the whole planetary motion with the sun as the center of the system.

I Law (Law of Orbits):

“Every planet moves in an elliptical orbit with the sun being at one of its foci”.

II Law (Law of Area):

“The radius vector drawn from the sun to the planet, sweeps equal areas in equal intervals of time. That, is the areal velocity is constant”.

III Law (Law of Period):

“The square of the time period of revolution of a planet around the sun is proportional to the cube of the semi-major axis of the elliptical orbit”.

Derivation of Kepler's Laws:

Consider a planet P, of mass m, moving under the force of gravitational attraction of the sun, of mass M. According to Newton's law of gravitation, the gravitational force exerted by the sun on the planet is given by,

$$F = \frac{-G\,M\,m}{r^2}\,\hat{r}$$

Negative sign indicates the force is acting towards the sun. In a small interval of time, the planet can be considered to move along a curve in a plane, with the sun being at the origin. Then the acceleration of the planet, in general is given by,

$$a = \left[\frac{d^2r}{dt^2} - r\left(\frac{d\theta}{dt}\right)^2\right]\hat{r} + \left[\frac{1}{r}\frac{d}{dt}\left(r^2\frac{d\theta}{dt}\right)\right]\hat{r}_\perp$$

$\left[\frac{d^2r}{dt^2} - r\left(\frac{d\theta}{dt}\right)^2\right]\hat{r}$ is the radial component of acceleration.

$\left[\frac{1}{r}\frac{d}{dt}\left(r^2\frac{d\theta}{dt}\right)\right]\hat{r}_\perp$ is the transverse component of acceleration.

Then, the force experienced by the planet, due to this acceleration, is given by,

$$F = ma = m\left\{\left[\frac{d^2r}{dt^2} - r\left(\frac{d\theta}{dt}\right)^2\right]\hat{r} + \left[\frac{1}{r}\frac{d}{dt}\left(r^2\frac{d\theta}{dt}\right)\right]\hat{r}_\perp\right\}$$

$$\frac{-G\,M\,m}{r^2}\,\hat{r} = m\left\{\left[\frac{d^2r}{dt^2} - r\left(\frac{d\theta}{dt}\right)^2\right]\hat{r} + \left[\frac{1}{r}\frac{d}{dt}\left(r^2\frac{d\theta}{dt}\right)\right]\hat{r}_\perp\right\}$$

$$\therefore\ \frac{-G\,M}{r^2}\,\hat{r} = \left[\frac{d^2r}{dt^2} - r\left(\frac{d\theta}{dt}\right)^2\right]\hat{r} + \left[\frac{1}{r}\frac{d}{dt}\left(r^2\frac{d\theta}{dt}\right)\right]\hat{r}_\perp$$

But, the gravitational force of attraction on the planet is radial and the transverse acceleration is zero.

$$\therefore\ \frac{-G\,M}{r^2}\,\hat{r} = \left[\frac{d^2r}{dt^2} - r\left(\frac{d\theta}{dt}\right)^2\right]\hat{r} \dots\dots\dots.(1)$$

$$\text{and } \left[\frac{1}{r}\frac{d}{dt}\left(r^2\frac{d\theta}{dt}\right)\right]\hat{r}_\perp = 0 \dots\dots\dots(2)$$

Derivation of Kepler's II Law:

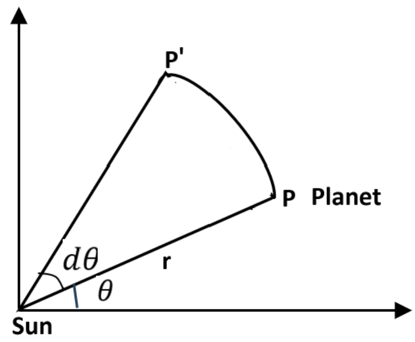
Consider transverse component of the acceleration,

$$\left[\frac{1}{r}\frac{d}{dt}\left(r^2\frac{d\theta}{dt}\right)\right]\hat{r}_\perp = 0$$

$$\text{which implies,} \left(r^2 \frac{d\theta}{dt}\right) = a\ constant$$

dividing and multiplying the LHS of the above equation by 2, we get,

$$2 \times \frac{1}{2}\left(r^2 \frac{d\theta}{dt}\right) = a\ constant \ldots\ldots\ldots (1)$$



suppose the planet moves from P to P' in a small interval of time dt, then, the area swept by the radius vector r is given by the area of the triangle SPP'

$$Area\ of\ the\ \Delta\ SPP' = \frac{1}{2} \times base \times height$$

$$= \frac{1}{2} \times r \times r\ d\theta$$

$$= \frac{1}{2}\ r^2\ d\theta$$

The areal velocity is given by,

$$h = \frac{Area\ of\ the\ \Delta\ SPP'}{dt} = \frac{1}{2} r^2 \frac{d\theta}{dt} \ldots\ldots\ldots (2)$$

comparing equation (1) and (2), we get $2\ h = constant$, or the areal velocity h of the planet is constant. This is the second law of Kepler.

Derivation of Kepler's I Law:

Consider magnitude of the force experienced by the planet due to acceleration (radial acceleration),

$$\frac{-G\ M}{r^2} = \left[\frac{d^2r}{dt^2} - r\left(\frac{d\theta}{dt}\right)^2\right] \ldots\ldots\ldots (1)$$

Multiplying both sides of the above equation by r^3, we get,

$$r^3\ \frac{d^2r}{dt^2} - r^4\left(\frac{d\theta}{dt}\right)^2 = -G\ M\ r$$

From Kepler's II law, we have,

$$2h = r^2\ \frac{d\theta}{dt} \ldots\ldots\ldots (2)$$

$$\therefore \quad r^3\ \frac{d^2r}{dt^2} - 4h^2 = -G\ M\ r \ldots\ldots\ldots (3)$$

$$\text{Let } r = \frac{1}{u}$$

$$\frac{dr}{d\theta} = \frac{-1}{u^2}\,\frac{du}{d\theta} \ldots\ldots\ldots\ldots(4)$$

$$\frac{dr}{dt} = \frac{dr}{d\theta}\,\frac{d\theta}{dt} \ldots\ldots\ldots..(5)$$

From equations (2), (4), and (5), we get,

$$\frac{dr}{dt} = \left(\frac{-1}{u^2}\,\frac{du}{d\theta}\right)\left(\frac{2h}{r^2}\right)$$

$$\frac{dr}{dt} = \left(-2h\,\frac{du}{d\theta}\right)$$

differentiating w.r.t. t , we get $\quad \frac{d^2r}{dt^2} = -2h\,\frac{d^2u}{d\theta^2}\,\frac{d\theta}{dt}$

$$\frac{d^2r}{dt^2} = -2h\,\frac{d^2u}{d\theta^2}\,.\,2hu^2$$

$$\frac{d^2r}{dt^2} = -4h^2u^2\,\frac{d^2u}{d\theta^2} \ldots\ldots\ldots.(6)$$

from equations (3) and (6), we get,

$$r^3\left(-4h^2u^2\,\frac{d^2u}{d\theta^2}\right) - 4h^2 = -G\,M\,r$$

$$\frac{4h^2u^2}{u^3}\,\frac{d^2u}{d\theta^2} + 4h^2 = \frac{G\,M}{u}$$

$$\frac{4h^2}{u}\,\frac{d^2u}{d\theta^2} + 4h^2 = \frac{G\,M}{u}$$

$$\frac{d^2u}{d\theta^2} + u = \frac{G\,M}{4h^2}$$

$$\frac{d^2u}{d\theta^2} + u - \frac{G\,M}{4h^2} = 0$$

The above equation can be written as,

$$\frac{d^2}{d\theta^2}\left(u - \frac{G\,M}{4h^2}\right) + \left(u - \frac{G\,M}{4h^2}\right) = 0$$ Since $\frac{G\,M}{4h^2}$ is constant

$$Let\ \left(u - \frac{G\,M}{4h^2}\right) = z$$

$$\frac{d^2z}{d\theta^2} + z = 0$$

This equation has a solution of the form, $z = A\,cos\theta$, where A is constant.

$$\therefore \quad u - \frac{G\,M}{4h^2} = A\,cos\theta$$

$$u = \frac{G\,M}{4h^2} + A\,cos\theta$$

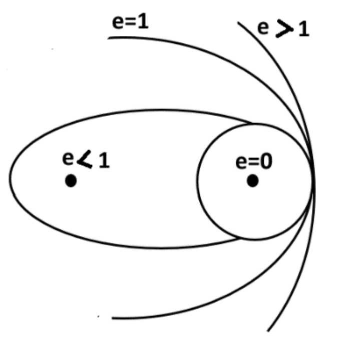
$$\frac{1}{r} = \frac{G\,M}{4h^2} + A\,cos\theta$$

$$\frac{4h^2}{G\,M\,r} = 1 + \frac{4h^2}{G\,M} A\,cos\theta$$

Let $\frac{4h^2}{G\,M} = l$ and $\frac{4h^2 A}{G\,M} = e$, then, the above equation becomes

$$\boldsymbol{\frac{l}{r} = 1 + e\,cos\theta} \;\ldots\ldots\ldots(7)$$

This is the equation of a conic section, where l is the semi latus rectum and e is the eccentricity. If $e < 1$ the above equation represents an ellipse with the sun being at on of its foci, which is the statement of Kepler's first law.



From equation (7) it is clear that the path of the planet is,

i. A circle when $e = 0$
ii. A parabola when $e = 1$
iii. An ellipse when $e < 1$
iv. A hyperbola when $e > 1$

Derivation of Kepler's III Law:

The path of a planet P, moving round the sun S, is an ellipse, with sun being at one of its foci. Let a, b and l be the semi-major axis, semi-minor axis and semi-latus rectum, respectively. Then, the relations between them in the case of an ellipse, are

$$b^2 = a^2(1 - e^2)$$

$$l = a(1 - e^2)$$

$e = \sqrt{1 - \frac{b^2}{a^2}}$

$latus\ rectum, 2\,l = \frac{2b^2}{a}$

$semi - latus\ rectum, l = \frac{b^2}{a}$

Therefore, from the two equations, we get

$$l = \frac{b^2}{a} = \frac{4h^2}{G\,M}$$

From Kepler's I Law derivation,
$l = \frac{4h^2}{G\,M}$

$$b^2 = \frac{4h^2 a}{G\,M} \dots\dots\dots (1)$$

If T is the period of revolution of the planet round the sun, then the areal velocity of the planet is given by

$$h = \frac{area\ of\ the\ ellipse}{period\ of\ revolution} = \frac{\pi ab}{T}$$

$$T = \frac{\pi ab}{h}$$

On squaring,

$$T^2 = \frac{\pi^2 a^2 b^2}{h^2} \dots\dots\dots..(2)$$

From equations (1) and (2),

$$T^2 = \frac{\pi^2 a^2}{h^2}\left(\frac{4h^2 a}{G\,M}\right)$$

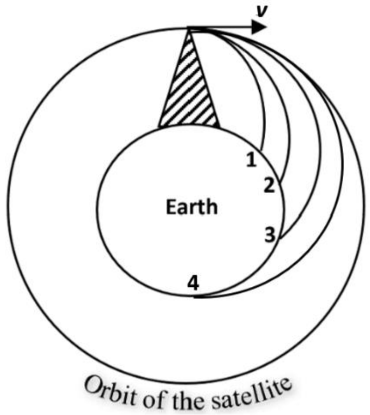
$$T^2 = \frac{4\,\pi^2 a^3}{G\,M}$$

$$\therefore\ \boldsymbol{T^2 \propto a^3}$$

That is, the square of the time period of revolution of a planet around the sun is proportional to the cube of the semi-major axis of the orbit. This is the statement of the Kepler's third law.

Satellites (Principle of satellite launching):

A solar system consists of objects called planets (like earth) revolving round a star (like sun). in addition to this, there may be other objects moving round the planets. These objects are called satellites. Moon is a satellite of the earth. Earth is a planet of the sun. apart from these natural satellites, there are man-made satellites which revolve around the planets are called artificial satellites.

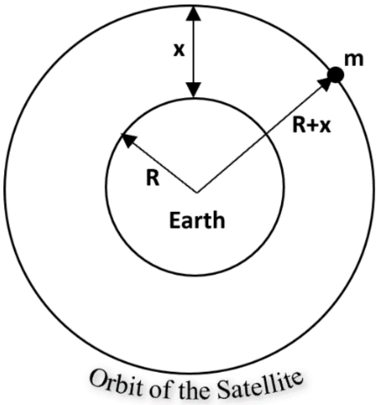


Newton argued if we imagine a mountain high enough so that it rises above earth's atmosphere as in figure. From the top of the mountain if we fire a bullet horizontally with some velocity. After some time, due to gravitational attraction the bullet will fall on the ground at position 1 at some distance from the foot of the mountain. If the bullet is fired with grater velocity it will fall at position 2 a bit farther than before if the velocity is further increased the bullet falls at position 3 still farther. For certain velocity the bullet will fall at the position 4, a point just opposite to the mountain on the globe. For velocity greater than this, the bullet will never hit the ground rather it will continue to fall freely under the gravity missing the earth each time and will continue to revolve round the earth in a circular orbit as moon does and become a satellite of the earth. Thus, the launching of an artificial satellite is done by means of multistage booster rockets. These rockets import very high velocities to overcome the force of gravity due to earth and air resistance.

Elements of satellite motion:

Two important quantities which govern the satellite motion are the ***orbital velocity*** and the ***time period*** of the satellite moving round the planet.

Expression for Orbital Velocity:



Suppose a satellite of mass 'm' has to be put into circular orbit around the earth at a height 'x' above its surface. Consider that earth is a sphere of mass 'M' and radius 'R'. then, radius of the orbit of the satellite will be $(R + x)$.

The gravitational force of attraction between the satellite and the earth will provide the necessary centripetal force to the satellite to move around the earth in the circular orbit.

$$\frac{G\,M\,m}{(R+x)^2} = \frac{mv^2}{(R+x)}$$

$$v^2 = \frac{G\,M}{R+x}$$

$$v = \sqrt{\frac{G\,M}{R+x}} \ldots\ldots (1)$$

The acceleration due to gravity 'g' on the surface of the earth is given by,

$$g = \frac{G\,M}{R^2} \qquad\qquad mg = \frac{GMm}{R^2}$$

$$G\,M = gR^2 \ldots\ldots (2)$$

From equations (1) and (2), we get,

$$v = \sqrt{\frac{gR^2}{R+x}} \ldots\ldots (3)$$

When the satellite is orbiting very close to the surface of the earth, then, $x = 0$. Thus, equation (3) becomes,

$$v = \sqrt{\frac{gR^2}{R}}$$

$$\boldsymbol{v = \sqrt{gR}}$$

This is the expression for the ***orbital velocity*** of the satellite. Substituting the values of g and R, the orbital velocity of the satellite is,

$$v = \sqrt{9.8 \times 6.4 \times 10^6} = 7.92\ Kms^{-1}$$

Therefore, velocity required to revolve just near the surface of the earth is $7.92\ Kms^{-1}$.

Time Period of Satellite:

The time period of a satellite is the time taken by it to go once around the earth.

$$T = \frac{Circumference\ of\ the\ orbit}{Orbital\ velocity}$$

$$T = \frac{2\,\pi\,(R+x)}{\sqrt{\frac{gR^2}{R+x}}}$$

$$T = 2\,\pi\sqrt{\frac{(R+x)^3}{gR^2}}$$

When the satellite revolves very close to the earth's surface $(x = 0)$, then,

$$T = 2\,\pi\sqrt{\frac{R^3}{gR^2}}$$

$$\boldsymbol{T = 2\,\pi\sqrt{\frac{R}{g}}}$$

This is the expression for time period of the satellite. Substituting the values of R and g in the above equation, time period of a satellite revolving close to the earth's surface is about 85 seconds.

Altitude of the Satellite:

The time period of the satellite is given by,

$$T = 2\,\pi\sqrt{\frac{(R+x)^3}{gR^2}}$$

$$T = 2\pi \sqrt{\frac{(R+x)^3}{G\ M}} \qquad\qquad mg = \frac{GMm}{R^2}$$

On squaring, we get,

$$T^2 = 4\pi^2 \frac{(R+x)^3}{G\ M}$$

$$(R+x)^3 = \frac{G\ M\ T^2}{4\ \pi^2}$$

$$(R+x) = \sqrt[3]{\frac{G\ M\ T^2}{4\ \pi^2}}$$

$$\boldsymbol{x = \sqrt[3]{\frac{G\ M\ T^2}{4\ \pi^2}} - R}$$

This is the expression for altitude of the satellite.

Escape Velocity:

When we throw a stone with some velocity it comes back after reaching a certain height. In general, a body projected upwards comes down to earth due to the gravitational pull of the earth on it. If we wish to project a body such that it never comes back, we should project the body with velocity which takes it beyond the gravitational field on the earth. This velocity is called velocity of escape or escape velocity. Thus, the escape velocity is defined as "***The minimum velocity with which an object has to be projected from the surface of the planet so that it escapes the planet's gravitational force of attraction***".

If 'm' be the mass of the body, 'M' that of earth and 'R', its radius, then the gravitational force acting on the body at a distance 'x' from the center of the earth is,

$$F = \frac{G\ M\ m}{x^2}$$

The work done by the body against the gravitational field of the earth in moving upwards through a distance dx is given by,

$$dW = \frac{G\ M\ m}{x^2}.dx \qquad\qquad Work\ done = Force \times\perp_r Distance$$

Therefore, total work done by the body in escaping away from the surface of the earth, that is, in moving away to an infinite distance from it, is given by

$$P.E = \int_R^\infty \frac{G\ M\ m}{x^2}.dx$$

$$P.E = G\ M\ m\left[-\frac{1}{x}\right]_R^\infty$$

$$P.E = G\ M\ m\ \left[\frac{1}{\infty} + \frac{1}{R}\right]$$

$$P.E = \frac{G\ M\ m}{R}$$

If v_e is the escape velocity then the K.E of the body $\frac{1}{2}mv_e^2$ must be equal to P.E of the body in escaping away from the earth.

$$\frac{1}{2}mv_e^2 = \frac{G\ M\ m}{R}$$

$$v_e^2 = \frac{2\ G\ M}{R}$$

$$v_e = \sqrt{\frac{2\ G\ M}{R}}$$

$$v_e = \sqrt{\frac{2\ g\ R^2}{R}} \qquad mg = \frac{GMm}{R^2}$$

$$\boldsymbol{v_e = \sqrt{2\ g\ R}}$$

Putting $g = 9.8\ ms^{-2}\ \ and\ \ R = 6.4 \times 10^6\ m$ in the above equation, we get, $v_e = 11.2\ Kms^{-1}$. That is the velocity with which a body should be projected so that it never comes back to the earth is $11.2\ Kms^{-1}$.

Geostationary Satellite:

Geostationary Satellite is defined as ***A satellite that revolves around the Earth in its equatorial plane with the same angular speed and in the same direction as the Earth rotates about its own axis.***

It is a satellite that revolves from West to East with a period of revolution equal to 24 hours in a circular orbit concentric and coplanar with the equatorial plane of the Earth. Because of the fact that their relative velocity with respect to Earth is zero, they appear to be stationary to an observer on Earth, and that is why are known as geostationary. They are also known as synchronous satellites because their angular velocity is the same as that of the Earth on its own axis. Angular Velocity of a geostationary satellite is **π/12 radians per hour.** The **height** of geostationary satellite from **earth's surface** is around **35,786 Km.** The Time Period of Geostationary satellites is **23 hours and 56 minutes 4 seconds** i.e. almost 1 day or 24 hours.

Necessary Conditions for a Geostationary Satellite:

A geostationary satellite must satisfy the following conditions.

- It should revolve in an orbit coplanar and concentric with the equatorial plane of the Earth.
- Its sense of rotation should be the same as that of the Earth *i.e.*, from west to east.
- Its period of revolution around the Earth should be exactly the same as that of the Earth on its own axis.

Geostationary Satellites Uses:

Some main uses are as follows.

- In studying the upper regions of the atmosphere.
- It is also used in forecasting weather.
- In determining the exact dimensions and shape of the Earth.
- Geostationary satellites have proven to be highly helpful for communication purposes. Due to the difference in curvature of the Earth, three uniformly spaced satellites are placed in orbit to establish communication between two locations.
- In studying meteorites.
- In studying the radiation of cosmic rays.

The Indian National Satellite System or INSAT, is a series of multipurpose geostationary satellites launched by the Indian Space Research Organisation (ISRO) to satisfy telecommunications, broadcasting, meteorology, and search and rescue operations. Commissioned in 1983, INSAT is the largest domestic communication system in the Indo-Pacific Region. It is a joint venture of the Department of Space, Department of Telecommunications, India Meteorological Department, All India Radio and Doordarshan. The overall coordination and management of INSAT system rests with the Secretary-level INSAT Coordination Committee.

INSAT Series	Launch Date	Launch Vehicle	Status
INSAT -1A	10 April 1982	Delta 3910 / PAM-D	Decommissioned (6 September 1982)
INSAT -1B	30 August 1983	U S Space Shuttle / PAM-D	Decommissioned (August 1993)
INSAT -1C	21 July 1988	Ariane 3	Decommissioned (2001)

INSAT -1D	9 July 1992	Delta 4925	Decommissioned (2001)
INSAT -2A	22 July 1993	Ariane 44L H10	Decommissioned (30 May 2002)
INSAT -2B	12 June 1990	Ariane 44L H10+	Decommissioned (1 July 2004)
INSAT -2C	6 December 1995	Ariane 44L H10-3	In Service
INSAT -2D	3 June 1997	Ariane 44L H10-3	Decommissioned (4 Oct, 1997)
INSAT -2DT	26 February 1992	Ariane 44L H10	Decommissioned (October 2004)
INSAT -2E	2 April 1999	Ariane 42P H10-3	In Service
INSAT -3A	9 April 2003	Ariane 5G	In Service
INSAT -3B	21 March 2000	Ariane 5G	In Service
INSAT -3C	23 January 2002	Ariane 42L H10-3	In Service
INSAT -3D	25 July 2013	Ariane 5 ECA	In Service
INSAT -3DR	8 September 2016	GSLV Mk II	In Service
INSAT -3DS	17 February 2024	GSLV Mk II	In Service

Remote Sensing Satellites:

Remote sensing is a process by which objects on the earth are sensed remotely by satellites. It acquires information about the earth's surface from distance far away, generally from an aircraft or satellite, polar orbits near the earth (about 800 km), with inclination between 95^0 and 105^0 with reference to the equator, are commonly used for remote sensing purpose. Geosynchronous orbit, about 36,000 km above the equator, is required for operating communications, weather and maritime satellites. Satellites have added a new dimension to geodesy, the science of surveying and mapping the earth's surface.

Applications:

- By revealing the gravitational anomalies of the earth, satellites have helped the scientists to outline the real shape of the earth.
- Remote sensing satellites have become accurate tools necessary to determine locations, distances and directions, which are helpful in fields like navigation, geophysical, prospecting etc.

- Remote sensing has helped in surveying crops, forests, land-use pattern, in mapping the state of the ocean.
- Remote sensing is of great help to the developing countries for proper management of their natural resources and develop themselves economically.
- It is also useful to avert or face with full preparedness for the natural calamities and save life and property.

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