Year 9 Indices – Content Summary, Part 4: Scientific Notation

- **Express numbers in scientific notation**

Scientific notation, also called 'standard form', is a consistent and convenient way to write either very large or very small numbers, allowing such numbers to be easily read and compared.

To understand how scientific notation works, we need to be familiar with our powers of ten and how these are used to express numbers in **index form**.

For example, the number '100' can be represented as '10 x 10' – that is, 10^2.

'1000' is '10 x 10 x 10', or 10^3.

The following table summarises the index notation format for numbers from 100 to 1 000 000 000 (1 billion):

Number *(Multiples of 10)*	Number in words	Number in Index Notation
100	One hundred	10^2
1 000	One thousand	10^3
10 000	Ten thousand	10^4
100 000	One hundred thousand	10^5
1 000 000	One million	10^6
10 000 000	Ten million	10^7
100 000 000	One hundred million	10^8
1 000 000 000	One billion	10^9

What do you notice about the **pattern** between the written number and its representation in index notation form?

If you looked closely, you may have picked up that the index number is the same number as the number of zeroes in the numerical form. For example, 'one-hundred million' (100 000 000) has 8 zeroes in it – and it is represented in index form as 10^8.

Scientific notation, however, is slightly different to index notation in that it is always expressed with **one decimal place to the left of the decimal**. We then 'raise' the remainder of the number to the appropriate power of ten using index notation. Thus,

$$41\ 400\ 000 = 4.14 \times 10^7$$

This is simply saying that if we multiply the number '4.14' by 10^7 (10 000 000 or 'ten million'), we will get the number 41 400 000 – which is our number expressed in its numerical form.

Scientific notation is used for very large numbers, as well as very small numbers. For example, Scientific Notation is often used to represent very large distances when measuring between planets and interstellar objects; it is also used when measuring minute sizes such as the size of virus cells or atomic level matter.

Let's start with large numbers.

See if you can convert the large intra-galactic distances in the table below into their index notation form by expressing them as a multiple of a power of ten.

The first two have been done for you:

Planet	Average Distance from Earth in km[2] – Numerical Form	Average Distance from Earth in km – Scientific Notation
Jupiter	628 730 000	**6.2873 x 10^8**
Mars	78 340 000	**7.834 x 10^7**
Mercury	91 691 000	
Neptune	4 351 400 000	
Saturn	1 275 000 000	
Uranus	2 724 000 000	
Venus	41 400 000	

Let's now look at very small numbers.

For example, an Ångstrom (Å) is a unit of length equal to 0.0000000001 of a metre, which is the approximate diameter of an atom.

Another way to express this is in fraction form (instead of decimal form):

0.0000000001 = $\frac{1}{10\,000\,000\,000}$ (of a metre). That's a *very* small faction!

Something you might pick up is that $\frac{1}{10\,000\,000\,000}$ is the reciprocal of 10 000 000 000 (or 'ten thousand million', or 'ten billion').

That is, $\frac{1}{10\,000\,000\,000}$ **is the reciprocal of 10^{10}**.

Can you remember how to express the reciprocal (ie, $\frac{1}{x}$ where x is the base number) of a number in it's index form?

That's right! It's simply the base number raised to the power of that same index in negative form! So,

$$0.0000000001 = \frac{1}{10\,000\,000\,000} = 1 \times 10^{-10}$$

Notice here that for Scientific Notation form of such small numbers, we still place a decimal point after the first non-zero digit and multiply by the appropriate power of 10. Only this time, the appropriate power will be a *negative* index!

Thus, for an atom of size 4.2 Å, or 0.00000000042 metres, we would write that the atom's size in Scientific Notation is

$$4.2 \times 10^{-10} \text{ metres.}$$

To convert a number from Scientific Notation into Numerical Form using a calculator, simply enter the prefix of the Sci Not number, then hit the 'x10^x' button, then enter the index (whether positive of negative).

For example, to find 3.2454 x 10^6, simply type in '3.2454', then hit the 'x10^x' button, then type in the index number, which is '6'. The answer '3 245 400' should result.

To display a number in its Scientific Notation form on a scientific calculator, you will need to use the 'SCI' (Scientific Notation) mode on your calculator. How to do this will vary depending on the make and model of your calculator (check your calculator instruction manual), however, once set, it should simply be a matter of entering the numerical form number, entering the number of 'significant figures' you'd like in your answer (e.g. a Sci Not number '3.45 x 10^5' has 3 significant figures, being the '3' and the '4' and '5'), and then hitting the 'equals' button (or some other conversion button, such as 'S⇔D').

The following YouTube clip may assist you with this process: **https://youtu.be/M1X_rnp12wo**

Of course, we can convert numbers expressed in Scientific Notation into Numerical form manually, simply by looking at the number in the index and counting over than number of decimal places – to the right if a positive index and the left if a negative index – to arrive at the numerical form number.

For example:

5.67 x 10^7   (This means *multiply* 5.67 by 10 000 000, and so count over 7 decimal places **to the right** from where it is in 5.67, because the index in this case is **positive**…)

= **56 700 000.**

Or:

6.8 x 10^{-5}   (This means *divide* 6.8 by 100 000, and so count over 5 decimal places **to the left** from where it is in 6.8, because the index in this case is **negative**…)

= **0.000068**

Your turn!

Use your textbook to explore some problems involving scientific notation.

Use your web browser and scientific calculator to investigate the following, and express the answers in both Numerical and Scientific Notation forms:

- The size of an average human blood cell in metres;

- The approximate mass of the Earth in kilograms;

- The distance between our galaxy (The Milky Way) and the Andromeda Nebula in light years;

- The size of an atomic nucleus for the Hydrogen element; and

- The number of human beings on Earth today.

Main References:

Brown, P., Evans, M., Gaudry, G., Hunt, D., McLaren, R., Pender, B. and Woolacott, B. (2011), *ICE-EM Mathematics, Year 9, Book 1*, Chapter 8. Cambridge University Press : Melbourne, Victoria.

Garrett, M. (2016), Mission to Mars: An AMSI Schools Rich Task in Number and Place Value. Accessed at https://calculate.org.au/2016/04/27/mission-mars-amsi-schools-rich-task-number-place-value/. Australian Mathematical Sciences Institute: Melbourne, VIC.