SUPPLEMENT 3B

6.4 General Strategy for Factoring Polynomials

Learning Objectives

By the end of this section, you will be able to:

- Recognize and use the appropriate method to factor a polynomial completely

Recognize and Use the Appropriate Method to Factor a Polynomial Completely

You have now become acquainted with all the methods of factoring that you will need in this course. The following chart summarizes all the factoring methods we have covered, and outlines a strategy you should use when factoring polynomials.

General Strategy for Factoring Polynomials

GCF

Binomial

- **Difference of Squares**
  $a^2 - b^2 = (a - b)(a + b)$
- **Sum of Squares**
  Sums of squares do not factor.
- **Sum of Cubes**
  $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$
- **Difference of Cubes**
  $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$

Trinomial

- $x^2 + bx + c$
  $(x \quad)(x \quad)$
- $ax^2 + bx + c$
  - **'a' and 'c' squares**
    $(a + b)^2 = a^2 + 2ab + b^2$
    $(a - b)^2 = a^2 - 2ab + b^2$
  - **'ac' method**

More than 3 terms

- grouping

HOW TO : : USE A GENERAL STRATEGY FOR FACTORING POLYNOMIALS.

Step 1. Is there a greatest common factor?
Factor it out.

Step 2. Is the polynomial a binomial, trinomial, or are there more than three terms?
If it is a binomial:

- Is it a sum?
  Of squares? Sums of squares do not factor.
  Of cubes? Use the sum of cubes pattern.
- Is it a difference?
  Of squares? Factor as the product of conjugates.
  Of cubes? Use the difference of cubes pattern.

If it is a trinomial:

- Is it of the form $x^2 + bx + c$? Undo FOIL.
- Is it of the form $ax^2 + bx + c$?
  If *a* and *c* are squares, check if it fits the trinomial square pattern.
  Use the trial and error or "ac" method.

If it has more than three terms:

- Use the grouping method.

Step 3. Check.
Is it factored completely?
Do the factors multiply back to the original polynomial?

Remember, a polynomial is completely factored if, other than monomials, its factors are prime!

EXAMPLE 6.35

Factor completely: $7x^3 - 21x^2 - 70x$.

Solution

	$7x^3 - 21x^2 - 70x$
Is there a GCF? Yes, $7x$.	
Factor out the GCF.	$7x\left(x^2 - 3x - 10\right)$
In the parentheses, is it a binomial, trinomial, or are there more terms?	
Trinomial with leading coefficient	
"Undo" FOIL.	$7x(x \quad)(x \quad)$
	$7x(x + 2)(x - 5)$
Is the expression factored completely? Yes.	
Neither binomial can be factored.	
Check your answer.	
Multiply.	

$$7x(x + 2)(x - 5)$$
$$7x\left(x^2 - 5x + 2x - 10\right)$$
$$7x\left(x^2 - 3x - 10\right)$$
$$7x^3 - 21x^2 - 70x \checkmark$$

> **TRY IT : :** 6.69 Factor completely: $8y^3 + 16y^2 - 24y$.

> **TRY IT : :** 6.70 Factor completely: $5y^3 - 15y^2 - 270y$.

Be careful when you are asked to factor a binomial as there are several options!

EXAMPLE 6.36

Factor completely: $24y^2 - 150$.

Solution

	$24y^2 - 150$
Is there a GCF? Yes, 6.	
Factor out the GCF.	$6\left(4y^2 - 25\right)$
In the parentheses, is it a binomial, trinomial or are there more than three terms? Binomial.	
Is it a sum? No.	
Is it a difference? Of squares or cubes? Yes, squares.	$6\left((2y)^2 - (5)^2\right)$
Write as a product of conjugates.	$6(2y - 5)(2y + 5)$

Is the expression factored completely?
Neither binomial can be factored.

Check:

Multiply.

$$6(2y - 5)(2y + 5)$$
$$6\left(4y^2 - 25\right)$$
$$24y^2 - 150 \checkmark$$

TRY IT : : 6.71 Factor completely: $16x^3 - 36x$.

TRY IT : : 6.72 Factor completely: $27y^2 - 48$.

The next example can be factored using several methods. Recognizing the trinomial squares pattern will make your work easier.

EXAMPLE 6.37

Factor completely: $4a^2 - 12ab + 9b^2$.

Solution

$$4a^2 - 12ab + 9b^2$$

Is there a GCF? No.
Is it a binomial, trinomial, or are there more terms?
Trinomial with $a \neq 1$. But the fir t term is a perfect square.

Is the last term a perfect square? Yes. $(2a)^2 - 12ab + (3b)^2$

Does it fit he pattern, $a^2 - 2ab + b^2$? Yes. $(2a)^2 \searrow \underset{-2(2a)(3b)}{-12ab +} \swarrow (3b)^2$

Write it as a square. $(2a - 3b)^2$

Is the expression factored completely? Yes.
The binomial cannot be factored.

Check your answer.

Multiply.

$$(2a - 3b)^2$$
$$(2a)^2 - 2 \cdot 2a \cdot 3b + (3b)^2$$
$$4a^2 - 12ab + 9b^2 \checkmark$$

TRY IT : : 6.73 Factor completely: $4x^2 + 20xy + 25y^2$.

TRY IT : : 6.74 Factor completely: $9x^2 - 24xy + 16y^2$.

Remember, sums of squares do not factor, but sums of cubes do!

EXAMPLE 6.38

Factor completely $12x^3y^2 + 75xy^2$.

Solution

$$12x^3y^2 + 75xy^2$$

Is there a GCF? Yes, $3xy^2$.

Factor out the GCF. $3xy^2\left(4x^2 + 25\right)$

In the parentheses, is it a binomial, trinomial, or are there more than three terms? Binomial.

Is it a sum? Of squares? Yes. Sums of squares are prime.

Is the expression factored completely? Yes.

Check:

Multiply.

$$3xy^2\left(4x^2 + 25\right)$$
$$12x^3y^2 + 75xy^2 \checkmark$$

TRY IT : : 6.75 Factor completely: $50x^3y + 72xy$.

TRY IT : : 6.76 Factor completely: $27xy^3 + 48xy$.

When using the sum or difference of cubes pattern, being careful with the signs.

EXAMPLE 6.39

Factor completely: $24x^3 + 81y^3$.

Solution

Is there a GCF? Yes, 3.	$24x^3 + 81y^3$
Factor it out.	$3(8x^3 + 27y^3)$
In the parentheses, is it a binomial, trinomial, of are there more than three terms? Binomial.	
Is it a sum or difference? Sum.	
Of squares or cubes? Sum of cubes.	$3\left(\overset{a^3}{(2x)^3} \overset{+}{+} \overset{b^3}{(3y)^3}\right)$
Write it using the sum of cubes pattern.	$3\left(\overset{a}{2x} \overset{+}{+} \overset{b}{3y}\right)\left(\overset{a^2}{(2x)^2} \overset{-}{-} \overset{ab}{2x \cdot 3y} \overset{+}{+} \overset{b^2}{(3y)^2}\right)$
Is the expression factored completely? Yes.	$3(2x + 3y)(4x^2 - 6xy + 9y^2)$
Check by multiplying.	

TRY IT : : 6.77 Factor completely: $250m^3 + 432n^3$.

TRY IT : : 6.78 Factor completely: $2p^3 + 54q^3$.

EXAMPLE 6.40

Factor completely: $3x^5 y - 48xy$.

Solution

	$3x^5y - 48xy$
Is there a GCF? Factor out $3xy$	$3xy\left(x^4 - 16\right)$
Is the binomial a sum or diffe ence? Of squares or cubes? Write it as a diffe ence of squares.	$3xy\left(\left(x^2\right)^2 - (4)^2\right)$
Factor it as a product of conjugates	$3xy\left(x^2 - 4\right)\left(x^2 + 4\right)$
The fir t binomial is again a diffe ence of squares.	$3xy\left((x)^2 - (2)^2\right)\left(x^2 + 4\right)$
Factor it as a product of conjugates.	$3xy(x - 2)(x + 2)\left(x^2 + 4\right)$

Is the expression factored completely? Yes.
Check your answer.
Multiply.

$$3xy(x - 2)(x + 2)\left(x^2 + 4\right)$$
$$3xy\left(x^2 - 4\right)\left(x^2 + 4\right)$$
$$3xy\left(x^4 - 16\right)$$
$$3x^5y - 48xy \checkmark$$

TRY IT : : 6.79 Factor completely: $4a^5b - 64ab$.

TRY IT : : 6.80 Factor completely: $7xy^5 - 7xy$.

EXAMPLE 6.41

Factor completely: $4x^2 + 8bx - 4ax - 8ab$.

Solution

	$4x^2 + 8bx - 4ax - 8ab$
Is there a GCF? Factor out the GCF, 4.	$4\left(x^2 + 2bx - ax - 2ab\right)$
There are four terms. Use grouping.	$4[x(x + 2b) - a(x + 2b)]$
	$4(x + 2b)(x - a)$

Is the expression factored completely? Yes.
Check your answer.
Multiply.

$$4(x + 2b)(x - a)$$
$$4\left(x^2 - ax + 2bx - 2ab\right)$$
$$4x^2 + 8bx - 4ax - 8ab \checkmark$$

TRY IT : : 6.81 Factor completely: $6x^2 - 12xc + 6bx - 12bc$.

TRY IT : : 6.82 Factor completely: $16x^2 + 24xy - 4x - 6y$.

Taking out the complete GCF in the first step will always make your work easier.

EXAMPLE 6.42

Factor completely: $40x^2y + 44xy - 24y$.

Solution

	$40x^2y + 44xy - 24y$
Is there a GCF? Factor out the GCF, $4y$.	$4y\left(10x^2 + 11x - 6\right)$
Factor the trinomial with $a \neq 1$.	$4y\left(10x^2 + 11x - 6\right)$
	$4y(5x - 2)(2x + 3)$

Is the expression factored completely? Yes.
Check your answer.
Multiply.

$$4y(5x - 2)(2x + 3)$$
$$4y\left(10x^2 + 11x - 6\right)$$
$$40x^2y + 44xy - 24y \checkmark$$

TRY IT : : 6.83 Factor completely: $4p^2q - 16pq + 12q$.

TRY IT : : 6.84 Factor completely: $6pq^2 - 9pq - 6p$.

When we have factored a polynomial with four terms, most often we separated it into two groups of two terms. Remember that we can also separate it into a trinomial and then one term.

EXAMPLE 6.43

Factor completely: $9x^2 - 12xy + 4y^2 - 49$.

Solution

	$9x^2 - 12xy + 4y^2 - 49$
Is there a GCF? No.	
With more than 3 terms, use grouping. Last 2 terms have no GCF. Try grouping fir t 3 terms.	$9x^2 - 12xy + 4y^2 - 49$
Factor the trinomial with $a \neq 1$. But the fir t term is a perfect square.	
Is the last term of the trinomial a perfect square? Yes.	$(3x)^2 - 12xy + (2y)^2 - 49$
Does the trinomial fit he pattern, $a^2 - 2ab + b^2$? Yes.	$(3x)^2 \searrow \underset{-2(3x)(2y)}{-12xy} + \swarrow (2y)^2 - 49$
Write the trinomial as a square.	$(3x - 2y)^2 - 49$
Is this binomial a sum or diffe ence? Of squares or cubes? Write it as a diffe ence of squares.	$(3x - 2y)^2 - 7^2$
Write it as a product of conjugates.	$((3x - 2y) - 7)((3x - 2y) + 7)$
	$(3x - 2y - 7)(3x - 2y + 7)$
Is the expression factored completely? Yes.	

Check your answer.

Multiply.

$$(3x - 2y - 7)(3x - 2y + 7)$$
$$9x^2 - 6xy - 21x - 6xy + 4y^2 + 14y + 21x - 14y - 49$$
$$9x^2 - 12xy + 4y^2 - 49 \checkmark$$

TRY IT : : 6.85 Factor completely: $4x^2 - 12xy + 9y^2 - 25$.

TRY IT : : 6.86 Factor completely: $16x^2 - 24xy + 9y^2 - 64$.

6.4 EXERCISES

Practice Makes Perfect

Recognize and Use the Appropriate Method to Factor a Polynomial Completely

In the following exercises, factor completely.

233. $2n^2 + 13n - 7$

235. $a^5 + 9a^3$

236. $75m^3 + 12m$

238. $49b^2 - 36a^2$

239. $8m^2 - 32$

241. $25w^2 - 60w + 36$

242. $49b^2 - 112b + 64$

244. $64x^2 + 16xy + y^2$

245. $7b^2 + 7b - 42$

247. $3x^4 y - 81xy$

248. $4x^5 y - 32x^2 y$

250. $m^4 - 81$

251. $5x^5 y^2 - 80xy^2$

253. $15pq - 15p + 12q - 12$

254. $12ab - 6a + 10b - 5$

256. $5q^2 - 15q - 90$

257. $4u^5 v + 4u^2 v^3$

259. $4c^2 + 20cd + 81d^2$

260. $25x^2 + 35xy + 49y^2$

262. $3v^4 - 768$

263. $36x^2 y + 15xy - 6y$

265. $8x^3 - 27y^3$

266. $64x^3 + 125y^3$

268. $y^6 + 1$

269. $9x^2 - 6xy + y^2 - 49$

271. $(3x + 1)^2 - 6(3x - 1) + 9$

272. $(4x - 5)^2 - 7(4x - 5) + 12$

6.5 Polynomial Equations

Learning Objectives

By the end of this section, you will be able to:

- Use the Zero Product Property
- Solve quadratic equations by factoring
- Solve equations with polynomial functions
- Solve applications modeled by polynomial equations

Be Prepared!

Before you get started, take this readiness quiz.

1. Solve: $5y - 3 = 0$.
2. Factor completely: $n^3 - 9n^2 - 22n$.
3. If $f(x) = 8x - 16$, find $f(3)$ and solve $f(x) = 0$.

We have spent considerable time learning how to factor polynomials. We will now look at polynomial equations and solve them using factoring, if possible.

A **polynomial equation** is an equation that contains a polynomial expression. The **degree of the polynomial equation** is the degree of the polynomial.

Polynomial Equation

A **polynomial equation** is an equation that contains a polynomial expression.

The **degree of the polynomial equation** is the degree of the polynomial.

We have already solved polynomial equations of degree one. Polynomial equations of degree one are linear equations are of the form $ax + b = c$.

We are now going to solve polynomial equations of degree two. A polynomial equation of degree two is called a **quadratic equation**. Listed below are some examples of quadratic equations:

$$x^2 + 5x + 6 = 0 \qquad 3y^2 + 4y = 10 \qquad 64u^2 - 81 = 0 \qquad n(n+1) = 42$$

The last equation doesn't appear to have the variable squared, but when we simplify the expression on the left we will get $n^2 + n$.

The general form of a quadratic equation is $ax^2 + bx + c = 0,$ with $a \neq 0$. (If $a = 0,$ then $0 \cdot x^2 = 0$ and we are left with no quadratic term.)

Quadratic Equation

An equation of the form $ax^2 + bx + c = 0$ is called a quadratic equation.

$$a,\ b,\ \text{and } c \text{ are real numbers and } a \neq 0$$

To solve quadratic equations we need methods different from the ones we used in solving linear equations. We will look at one method here and then several others in a later chapter.

Use the Zero Product Property

We will first solve some quadratic equations by using the **Zero Product Property**. The Zero Product Property says that if the product of two quantities is zero, then at least one of the quantities is zero. The only way to get a product equal to zero is to multiply by zero itself.

Zero Product Property

If $a \cdot b = 0,$ then either $a = 0$ or $b = 0$ or both.

We will now use the Zero Product Property, to solve a quadratic equation.

EXAMPLE 6.44 HOW TO SOLVE A QUADRATIC EQUATION USING THE ZERO PRODUCT PROPERTY

Solve: $(5n - 2)(6n - 1) = 0.$

Solution

Step 1. Set each factor equal to zero.	The product equals zero, so at least one factor must equal zero.	$(5n - 2)(6n - 1) = 0$ $5n - 2 = 0$ or $6n - 1 = 0$
Step 2. Solve the linear equations.	Solve each equation.	$n = \frac{2}{5}$ $\quad n = \frac{1}{6}$
Step 3. Check.	Substitute each solution separately into the original equation.	$n = \frac{2}{5}$ $(5n - 2)(6n - 1) = 0$ $\left(5 \cdot \frac{2}{5} - 2\right)\left(6 \cdot \frac{2}{5} - 1\right) \stackrel{?}{=} 0$ $(2 - 2)\left(\frac{12}{5} - 1\right) \stackrel{?}{=} 0$ $0 \cdot \frac{7}{5} \stackrel{?}{=} 0$ $0 = 0$ ✓ $n = \frac{1}{6}$ $(5n - 2)(6n - 1) = 0$ $\left(5 \cdot \frac{1}{6} - 2\right)\left(6 \cdot \frac{1}{6} - 1\right) \stackrel{?}{=} 0$ $\left(\frac{5}{6} - \frac{12}{6}\right)(1 - 1) \stackrel{?}{=} 0$ $\left(-\frac{7}{6}\right)(0) \stackrel{?}{=} 0$ $0 = 0$ ✓

TRY IT : : 6.87 Solve: $(3m - 2)(2m + 1) = 0.$

TRY IT : : 6.88 Solve: $(4p + 3)(4p - 3) = 0.$

HOW TO : : USE THE ZERO PRODUCT PROPERTY.

Step 1. Set each factor equal to zero.
Step 2. Solve the linear equations.
Step 3. Check.

Solve Quadratic Equations by Factoring

The Zero Product Property works very nicely to solve quadratic equations. The quadratic equation must be factored, with zero isolated on one side. So we be sure to start with the quadratic equation in standard form, $ax^2 + bx + c = 0$. Then we factor the expression on the left.

EXAMPLE 6.45 HOW TO SOLVE A QUADRATIC EQUATION BY FACTORING

Solve: $2y^2 = 13y + 45$.

Solution

Step 1. Write the quadratic equation in standard form, $ax^2 + bx + c = 0$.	Write the equation in standard form.	$2y^2 = 13y + 45$ $2y^2 - 13y - 45 = 0$
Step 2. Factor the quadratic expression.	Factor $2y^2 - 13y + 45$ $(2y + 5)(y - 9)$	$(2y + 5)(y - 9) = 0$
Step 3. Use the Zero Product Property.	Set each factor equal to zero. We have two linear equations.	$2y + 5 = 0 \quad y - 9 = 0$
Step 4. Solve the linear equations.		$y = -\frac{5}{2} \quad y = 9$
Step 5. Check. Substitute each solution separately into the original equation.	Substitute each solution separately into the original equation.	$y = -\frac{5}{2}$ $2y^2 = 13y + 45$ $2\left(-\frac{5}{2}\right)^2 \stackrel{?}{=} 13\left(-\frac{5}{2}\right) + 45$ $2\left(\frac{25}{4}\right) \stackrel{?}{=} \left(-\frac{65}{2}\right) + \frac{90}{2}$ $\frac{25}{2} = \frac{25}{2}$ ✓ $y = 9$ $2y^2 = 13y + 45$ $2(9)^2 \stackrel{?}{=} 13(9) + 45$ $2(81) \stackrel{?}{=} 117 + 45$ $162 = 162$ ✓

TRY IT : : 6.89 Solve: $3c^2 = 10c - 8$.

TRY IT : : 6.90 Solve: $2d^2 - 5d = 3$.

HOW TO : : SOLVE A QUADRATIC EQUATION BY FACTORING.

Step 1. Write the quadratic equation in standard form, $ax^2 + bx + c = 0$.
Step 2. Factor the quadratic expression.
Step 3. Use the Zero Product Property.
Step 4. Solve the linear equations.
Step 5. Check. Substitute each solution separately into the original equation.

Before we factor, we must make sure the quadratic equation is in standard form.

Solving quadratic equations by factoring will make use of all the factoring techniques you have learned in this chapter! Do you recognize the special product pattern in the next example?

EXAMPLE 6.46

Solve: $169q^2 = 49$.

Solution

	$169x^2 = 49$
Write the quadratic equation in standard form.	$169x^2 - 49 = 0$
Factor. It is a diffe ence of squares.	$(13x - 7)(13x + 7) = 0$
Use the Zero Product Property to set each factor to 0.	$13x - 7 = 0 \qquad 13x + 7 = 0$
Solve each equation.	$13x = 7 \qquad 13x = -7$
	$x = \frac{7}{13} \qquad x = -\frac{7}{13}$

Check:

We leave the check up to you.

TRY IT : : 6.91 Solve: $25p^2 = 49$.

TRY IT : : 6.92 Solve: $36x^2 = 121$.

In the next example, the left side of the equation is factored, but the right side is not zero. In order to use the Zero Product Property, one side of the equation must be zero. We'll multiply the factors and then write the equation in standard form.

EXAMPLE 6.47

Solve: $(3x - 8)(x - 1) = 3x$.

Solution

	$(3x - 8)(x - 1) = 3x$
Multiply the binomials.	$3x^2 - 11x + 8 = 3x$
Write the quadratic equation in standard form.	$3x^2 - 14x + 8 = 0$
Factor the trinomial.	$(3x - 2)(x - 4) = 0$
Use the Zero Product Property to set each factor to 0.	$3x - 2 = 0 \qquad x - 4 = 0$
Solve each equation.	$3x = 2 \qquad x = 4$
	$x = \frac{2}{3}$
Check your answers.	The check is left to you.

TRY IT : : 6.93 Solve: $(2m + 1)(m + 3) = 12m$.

TRY IT : : 6.94 Solve: $(k + 1)(k - 1) = 8$.

In the next example, when we factor the quadratic equation we will get three factors. However the first factor is a constant. We know that factor cannot equal 0.

EXAMPLE 6.48

Solve: $3x^2 = 12x + 63$.

Solution

	$3x^2 = 12x + 63$
Write the quadratic equation in standard form.	$3x^2 - 12x - 63 = 0$
Factor the greatest common factor fir t.	$3\left(x^2 - 4x - 21\right) = 0$
Factor the trinomial.	$3(x - 7)(x + 3) = 0$
Use the Zero Product Property to set each factor to 0.	$3 \neq 0 \quad x - 7 = 0 \quad x + 3 = 0$
Solve each equation.	$3 \neq 0 \quad x = 7 \quad x = -3$
Check your answers.	The check is left to you.

TRY IT : : 6.95 Solve: $18a^2 - 30 = -33a$.

TRY IT : : 6.96 Solve: $123b = -6 - 60b^2$.

The Zero Product Property also applies to the product of three or more factors. If the product is zero, at least one of the factors must be zero. We can solve some equations of degree greater than two by using the Zero Product Property, just like we solved quadratic equations.

EXAMPLE 6.49

Solve: $9m^3 + 100m = 60m^2$.

Solution

	$9m^3 + 100m = 60m^2$
Bring all the terms to one side so that the other side is zero.	$9m^3 - 60m^2 + 100m = 0$
Factor the greatest common factor fir t.	$m\left(9m^2 - 60m + 100\right) = 0$
Factor the trinomial.	$m(3m - 10)(3m - 10) = 0$
Use the Zero Product Property to set each factor to 0.	$m = 0 \quad 3m - 10 = 0 \quad 3m - 10 = 0$
Solve each equation.	$m = 0 \quad m = \frac{10}{3} \quad m = \frac{10}{3}$
Check your answers.	The check is left to you.

TRY IT : : 6.97 Solve: $8x^3 = 24x^2 - 18x$.

TRY IT : : 6.98 Solve: $16y^2 = 32y^3 + 2y$.

Solve Equations with Polynomial Functions

As our study of polynomial functions continues, it will often be important to know when the function will have a certain value or what points lie on the graph of the function. Our work with the Zero Product Property will be help us find these answers.

EXAMPLE 6.50

For the function $f(x) = x^2 + 2x - 2$,

ⓐ find *x* when $f(x) = 6$

ⓑ find two points that lie on the graph of the function.

6.5 EXERCISES

Practice Makes Perfect

Use the Zero Product Property

In the following exercises, solve.

277. $(3a-10)(2a-7)=0$

279. $6m(12m-5)=0$

280. $2x(6x-3)=0$

282. $(3y+5)^2=0$

Solve Quadratic Equations by Factoring

In the following exercises, solve.

283. $5a^2-26a=24$

285. $4m^2=17m-15$

286. $n^2=5-6n$

288. $12b^2-15b=-9b$

289. $49m^2=144$

291. $16y^2=81$

292. $64p^2=225$

294. $100y^2=9$

295. $(x+6)(x-3)=-8$

297. $(2x+1)(x-3)=-4x$

298. $(y-3)(y+2)=4y$

300. $(2y-3)(3y-1)=8y$

301. $20x^2-60x=-45$

303. $15x^2-10x=40$

304. $14y^2-77y=-35$

306. $16y^2+12=-32x$

307. $16p^3=24p^2+9p$

309. $2x^3+72x=24x^2$

310. $3y^3+48y=24y^2$

312. $2y^3+2y^2=12y$

Solve Equations with Polynomial Functions

In the following exercises, solve.

313. For the function, $f(x)=x^2-8x+8$, ⓐ find when $f(x)=-4$ ⓑ Use this information to find two points that lie on the graph of the function.

314. For the function, $f(x)=x^2+11x+20$, ⓐ find when $f(x)=-8$ ⓑ Use this information to find two points that lie on the graph of the function.